Static Decouplers for Control of Multivariable Processes

Jietae Lee and Dong Hyun Kim
Dept. of Chemical Engineering, Kyungpook National University, Taegu 702-701, Korea

Thomas F. Edgar
Dept. of Chemical Engineering, University of Texas, Austin, TX 78712

DOI 10.1002/aic.10520
Published online July 7, 2005 in Wiley InterScience (www.interscience.wiley.com).

Decouplers can be used to improve performances of multiloop control systems by compensating for process interactions, but they are sensitive to process changes and require fairly detailed process models, which are hard to obtain. These disadvantages of decouplers hinder their successful applications in industry. On the other hand, static decouplers can be designed from process steady-state gains, which are easier to obtain and can be tuned in the field. However, static decouplers can introduce undesirable effects on loop interactions at high frequencies for some processes. Here it is shown that static decouplers are better applied to the integral control modes for such processes. Because the integral mode is dominant at low frequencies and its magnitude decreases as frequency increases, the static decoupler implemented only on the integral mode will not disturb the frequency response of a control system at high frequencies. A simple antireset windup scheme, guaranteeing given control performances under process input saturations or failures, is provided. For online tuning of the proposed control system, a closed-loop identification method to find process steady-state gains is also presented. *AIChE J,* 51: 2712–2720, 2005
Keywords: multiloop control, integral mode with a static decoupler, antireset windup, closed-loop identification

Introduction

Chemical processes are typically multi-input/multi-output (MIMO) systems. Although model-based multivariable controllers have been successfully applied, performance degradations can occur because of the lack of simple maintenance procedures to cope with poor process models and changing process environments. The control structures are complex and operators are unable to adjust them in the field when poor control performances occur as a result of process changes. On the other hand, multiloop control systems using multiple single-input/single-output (SISO) controllers remain the standard in most industries because of their simplicity and the ability to field-tune them for their performance improvements. The control performance of a well-tuned multiloop control system can be better than that of a loosely tuned multivariable control system. Because its best achievable performance is limited, a method to improve closed-loop performance of a multiloop control system without sacrificing the simplicity of a multiloop control system is investigated here.

As a straightforward extension of SISO proportional-integral/proportional-integral-derivative (PI/PID) controllers, multivariable PI/PID controllers by Koivo and Pohjolainen[1] and control systems designed by frequency response methods[2] used simple PI/PID control structures. Tan et al.[3] proposed a method to design multivariable PI/PID controllers by applying the Taylor series expansion technique to recent robust multivariable controllers, reporting promising control performances.

Correspondence concerning this article should be addressed to J. Lee at jtlee@knu.ac.kr.

However, their gain matrices for proportional, integral, and derivative modes are all full, requiring detailed process models to design, and field-tuning them is not easy.

Process interactions degrade performances of multiloop control systems. To improve the closed-loop performances of multiloop control systems, decouplers can be used. Dynamic decouplers compensate for process interactions and result in decoupled responses of controlled variables.[4,5] Decouplers have been well established in the field of distillation control and almost all textbooks for process control deal with them.[6,7] Arkun et al.[8] applied a robustness analysis procedure based on the singular value to distillation control systems with decouplers and concluded that ideal decoupling cannot be robust to modeling errors. Wade[9] and Waller et al.[10] showed that implementational obstacles such as realizability, bumpless switches between manual and automatic modes, and input saturations can be removed, remarking that their possibilities and advantages have been neglected in other fields of control.

Dynamic decouplers also require detailed process models. To avoid the requirement of detailed process models, static decouplers using the inverse of process steady-state gains can be used. Astrom et al.[11] showed that static decouplers are very effective for some processes when very fast controls are not required. Although static decouplers are simple to design and adjust in the field, they do not always provide better closed-loop performances. Static decouplers can cause undesirable effects on high-frequency responses for some processes. In this paper we show that static decouplers should be applied only to the integral modes for such processes. Frequency-dependent relative gain arrays[12] are used to determine whether static decouplers applied only on the integral modes are recommended. It is shown that static decouplers significantly improve control performances, whereas their stability robustness is very similar to that of multiloop control systems. To implement static decouplers, steady-state process gains are required and methods to find them are also investigated.

Process inputs usually have physical limits. When some inputs are saturated to their limits, those inputs lose their control abilities. Loop failure tolerance is a condition for control systems to remain stable under such situations. For closed-loop performance as well as the loop failure tolerance, an antireset windup scheme can be used. A simple antireset windup scheme for the proposed control system having integral mode with the static decoupler is proposed and it is shown that it guarantees a given control performance under process input saturations or failures.

Integral Mode with the Static Decoupler

Consider a process of n inputs and n outputs whose transfer function is

$$G(s) = \{g_{ij}(s),\ i = 1, n,\ j = 1, n\}$$

To improve control performances of multiloop controllers [$C(s) = K_I/s + K_C$, where K_I and K_C are diagonal], decouplers are considered. Although a dynamic decoupler can compensate for process interactions, it needs a rather detailed process model and its performance can be poor for an inaccurate process model and changing process conditions. Instead of dynamic decouplers, static decouplers designed from steady-state process gains can be used as follows

$$C(s) = G^{-1}(0)\left(\frac{1}{s}\tilde{K}_I + \tilde{K}_c\right) = \frac{1}{s}G^{-1}(0)\begin{bmatrix} \tilde{k}_{I1} & & 0 \\ & \cdots & \\ 0 & & \tilde{k}_{In} \end{bmatrix} + G^{-1}(0)\begin{bmatrix} \tilde{k}_{C1} & & 0 \\ & \cdots & \\ 0 & & \tilde{k}_{Cn} \end{bmatrix} \quad (1)$$

The static decoupler $G^{-1}(0)$ reduces process interactions at frequencies near zero. However, for some processes, it can introduce undesirable effects at high frequencies. Typically diagonal elements of the process transfer function are faster than off-diagonal elements. As frequency increases, magnitudes of off-diagonal elements decrease faster than those of diagonal elements and, consequently, interactions for such processes become smaller. However, when static decouplers are applied to such processes, off-diagonal elements of the decoupled process $G(s)G^{-1}(0)$ contain faster elements and their magnitudes relative to those of diagonal elements will not decrease, resulting in higher process interactions at high frequencies.

There are many realistic process models for which the static decouplers can have negative effects on process interactions at high frequencies. For example, consider the Wood–Berry column model[13]

$$G(s) = \begin{bmatrix} \dfrac{12.8\exp(-s)}{16.7s + 1} & \dfrac{-18.9\exp(-3s)}{21s + 1} \\ \dfrac{6.6\exp(-7s)}{10.9s + 1} & \dfrac{-19.4\exp(-3s)}{14.4s + 1} \end{bmatrix} \quad (2)$$

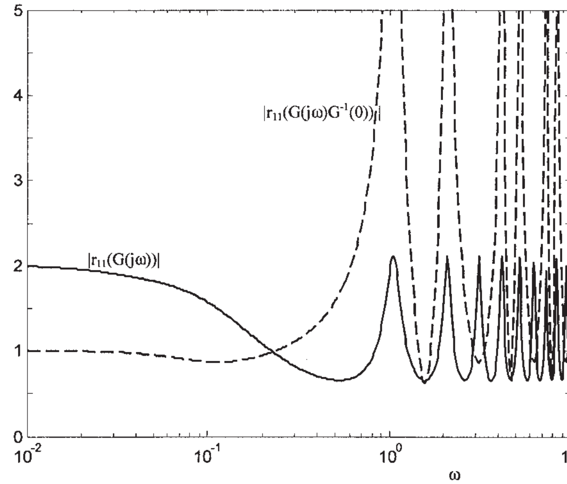

Figure 1. Frequency-dependent relative gain arrays.

Solid line is for $G(j\omega)$ and dashed line is for $G(j\omega)G^{-1}(0)$.

Figure 1 shows the absolute value of the (1, 1) element of relative gain array[12] of $G(j\omega)$ and that of the decoupled process $G(j\omega)G^{-1}(0)$, where ω is the angular frequency. The relative gain array (RGA) of $G(j\omega)$ can be calculated by

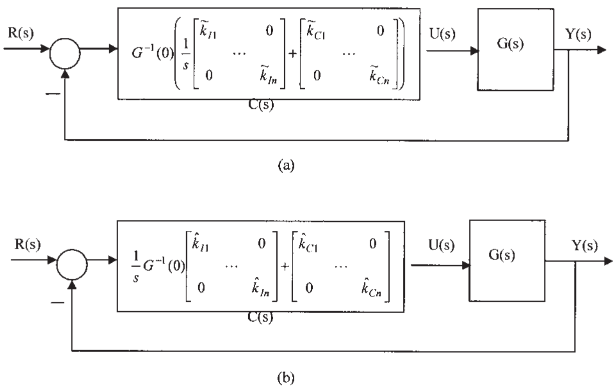

Figure 2. Static decouplers applied to (a) multiloop controller and (b) integral mode only.

$$R[G(j\omega)] = G(j\omega)^{-T} \circ G(j\omega) \tag{3}$$

where $\circ$ is the element-by-element multiplication. At low frequencies, the (1, 1) element of the relative gain array of a decoupled process is near unity and the static decoupler effectively removes the process interaction. However, the (1, 1) element of the relative gain array of the decoupled process becomes very large as frequency increases. This means that process interactions become worse as a result of the static decoupler. Large values in the relative gain array imply that the closed-loop system can be very sensitive to process uncertainties.[14,15]

To avoid the disadvantage that a static decoupler causes undesirable effects on reducing process interactions at high frequencies, we propose a control system where the static decoupler is applied for only the integral mode, as shown in Figure 2b

$$C(s) = \frac{1}{s}\, G^{-1}(0)\hat{K}_I + \hat{K}_c = \frac{1}{s}\, G^{-1}(0)\begin{bmatrix} \hat{k}_{I1} & & 0 \\ & \cdots & \\ 0 & & \hat{k}_{In} \end{bmatrix} + \begin{bmatrix} \hat{k}_{C1} & & 0 \\ & \cdots & \\ 0 & & \hat{k}_{Cn} \end{bmatrix} \tag{4}$$

Qualitatively, the integral mode $G^{-1}(0)\hat{K}_I/s$ is dominant at low frequencies and its magnitude will decrease as the frequency increases. Thus the static decoupler is effective only at low frequencies and its undesirable behavior at high frequencies will be avoided. Performances of control systems with the integral modes having static decouplers are investigated.

The concept where a static decoupler is implemented on the integral mode can be applied to the multiloop control system using a proportional and filtered integral (PFI) controller.[16] Each PFI controller is given as $c_i(s) = (k_{Ii}/s)/(\tau_{Fi}s + 1) + k_{Ci}$ and, because the magnitude of its integral term decreases more rapidly as the frequency increases, a static decoupler implemented on the integral mode may be more effective than that of Eq. 4.

Design Method

Case I

When well-tuned multiloop control systems are already given as $C(s) = K_I/s + K_C$, we can improve their performances by adjusting the integral terms as

$$C(s) = \frac{1}{s}\, G^{-1}(0)\mathrm{diag}\{k_{Ii}/[G^{-1}(0)]_{ii}\} + K_C \tag{5}$$

where $[G^{-1}(0)]_{ii}$ is the (i, i) element of $G^{-1}(0)$. Here diag(m_i) means a diagonal matrix whose ith element is m_i and diag(M) for a square matrix M means a diagonal matrix whose diagonal elements are those of M. As shown in the definition of relative gain array, $[G^{-1}(0)]_{ii}$ is the inverse of the effective process gain between the ith input and ith output when other loops are all controlled through controllers having integral actions. Because the steady-state process gain between the ith input and ith output is 1 when $G^{-1}(0)$ is introduced, we scale the integral term by its effective process gain before applying the static decoupler.

Case II

When multiloop controllers are not given, we design control systems from process models. First design a multiloop control system and then its integral term is modified. For a multiloop control system, a recent analytic method in Lee et al.[17] is used. The method finds PI controllers satisfying approximately

$$\mathrm{diag}\{[I + G(s)C(s)]^{-1}G(s)C(s)\} \approx \mathrm{diag}\left(\frac{g_{ii}^{+}(s)}{(\lambda_i s + 1)^{r_i}}\right) \tag{6}$$

where $g_{ii}^{+}(s)$ and r_i are the nonminimum-phase part and relative order of $g_{ii}(s)$, respectively, and λ_i is the design parameter representing the closed-loop time constant. By applying the desired closed-loop response method[18] to the diagonal elements of $G(s)$, we obtain

$$c_i(s) = \frac{1}{g_{ii}(s)}\frac{\bar{h}_i(s)}{1 - \bar{h}_i(s)} = \frac{k_{oi}}{s} + k_{1i} + k_{2i}s + \cdots$$

$$\bar{h}_i(s) = \frac{g_{ii}^{+}(s)}{(\lambda_i s + 1)^{r_i}} \tag{7}$$

By truncating Eq. 7 and adjusting the integral part to compensate for the gain change resulting from closing loops, Lee et al.[17] obtained a multiloop controller as

$$C(s) = \frac{1}{s}\, K_I + K_C = \frac{1}{s}\,\mathrm{diag}\{k_{0i}g_{ii}(0)[G^{-1}(0)]_{ii}\} + \mathrm{diag}(k_{1i}) \tag{8}$$

Because the integral part is dominant at low frequencies, this modification approximates Eq. 6 well at low frequencies. The proportional part is not adjusted because it is dominant at high frequencies and we can approximate $H(s)$ as $H(s) = [I + G(s)C(s)]^{-1}G(s)C(s) \approx G(s)C(s)$ at high frequencies. The multiloop control system designed analytically through Eqs. 7 and

Table 1. Control Systems Compared

Process	Method	$C(s)$
Wood–Berry column	Biggest log-modulus tuning[24]	$\begin{bmatrix} 0.375(1+1/8.29s) & 0 \\ 0 & -0.075(1+1/23.6s) \end{bmatrix}$
	Desired closed-loop response method[17] $\lambda = \{5, 5\}$	$\begin{bmatrix} 0.219(1+1/8.353s) & 0 \\ 0 & -0.0964(1+1/7.45s) \end{bmatrix}$
	Proposed $\lambda = \{5, 5\}$	$\frac{1}{s}\begin{bmatrix} 0.0262 & -0.0191 \\ 0.0089 & -0.0129 \end{bmatrix} + \begin{bmatrix} 0.219 & 0 \\ 0 & -0.0964 \end{bmatrix}$
Ogunnaike–Ray column	Biggest log-modulus tuning[24]	$\begin{bmatrix} 1.51(1+1/16.4s) & 0 & 0 \\ 0 & -0.295(1+1/18s) & 0 \\ 0 & 0 & 2.63(1+1/6.61s) \end{bmatrix}$
	Desired closed-loop response method $\lambda = \{15, 15, 5\}$	$\begin{bmatrix} 0.593(1+1/3.43s) & 0 & 0 \\ 0 & -0.124(1+1/2.88s) & 0 \\ 0 & 0 & 2.14(1+1/7.62s) \end{bmatrix}$
	Proposed $\lambda = \{15, 15, 5\}$	$\frac{1}{s}\begin{bmatrix} 0.1729 & -0.0323 & 0.0017 \\ 0.0672 & -0.0430 & -0.0004 \\ 3.3209 & 0.9920 & 0.2806 \end{bmatrix} + \begin{bmatrix} 0.5933 & 0 & 0 \\ 0 & -0.1236 & 0 \\ 0 & 0 & 2.1386 \end{bmatrix}$

8 shows excellent closed-loop performances for various processes when the λ_i design parameters are chosen not to be too small.

By applying the technique in Case I to the multiloop controller Eq. 8, we obtain

$$C(s) = \frac{1}{s} G^{-1}(0)\hat{K}_I + \hat{K}_C = \frac{1}{s} G^{-1}(0)\mathrm{diag}\{k_{Ii}/[G^{-1}(0)]_{ii}\} + K_C$$

$$= \frac{1}{s} G^{-1}(0)\mathrm{diag}[k_{0i}g_{ii}(0)] + K_C \qquad (9)$$

Gain changes arising from loop closing are compensated by a full matrix of $G^{-1}(0)\mathrm{diag}[g_{ii}(0)]$ in Eq. 9, whereas controller 8 uses a diagonal matrix of $\mathrm{diag}\{g_{ii}(0)[G^{-1}(0)]_{ii}\}$ for this purpose.[17] The success of controller 8 should carry over to the proposed controller 9.

Note that $\hat{K}_I = \mathrm{diag}[k_{0i}g_{ii}(0)]$ and $\hat{K}_C = K_C$ depend only on diagonal elements of $G(s)$. Thus they need not be redesigned to control principal subsystems of $G(s)$, where some input–output pairs are removed.

Performance and Dynamic Interaction Measure

A static decoupler implemented on the integral mode can improve the closed-loop performances at low frequencies. We compare control systems for the Wood–Berry column of Eq. 2 using a multiloop controller designed by Eq. 8 with $\lambda_1 = \lambda_2 = 5$ and the proposed controller 9 having an integral mode with the static decoupler. Detailed controller parameters are given in Table 1. Figure 3 shows the largest and smallest singular values of sensitivity function matrices and complementary sensitivity function matrices

$$S(s) = [I + G(s)C(s)]^{-1}$$

$$H(s) = I - S(s) = [I + G(s)C(s)]^{-1}G(s)C(s) \qquad (10)$$

We can see that the static decoupler implemented on the integral mode decreases the largest singular value $\sigma_{\max}[S(j\omega)]$ of sensitivity function matrix at low frequencies and increases the smallest singular value $\sigma_{\min}[H(j\omega)]$ of the complementary sensitivity function matrix. These imply that the control system can quickly track set-point changes while suppressing low-frequency disturbances.[2] Because the peak value of the largest singular value of the complementary sensitivity function matrix of the control system having integral mode with the static decoupler is less than that of the multiloop control system, the control system with the proposed controller 9 will be less oscillatory. At high frequencies, both control systems have

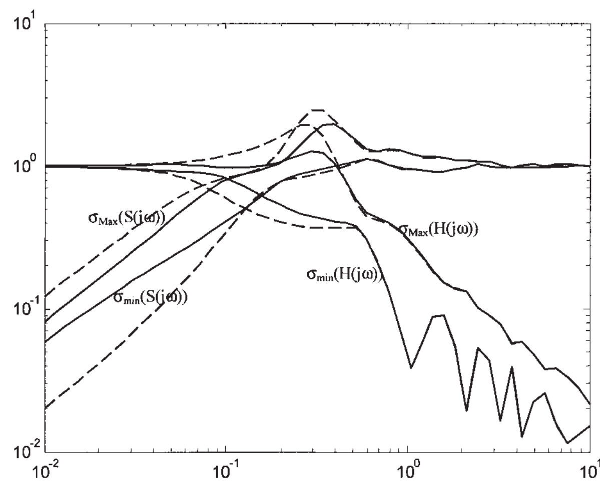

Figure 3. Maximum and minimum singular values of sensitivity and complementary sensitivity function matrices.

Solid line is for the multiloop control system and dashed line is for the proposed control system with integral mode having the static decoupler.

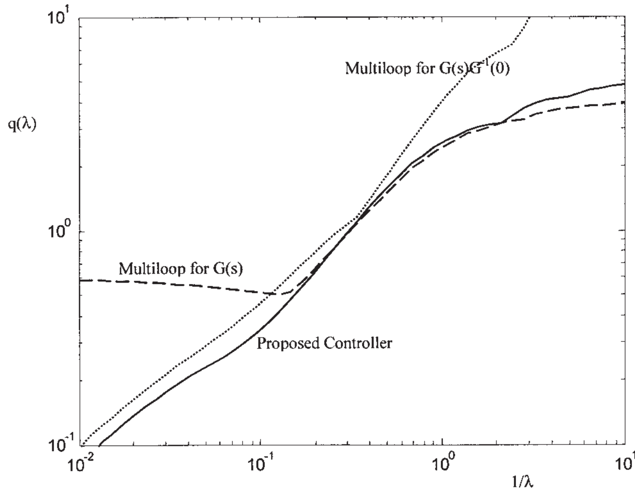

Figure 4. Dynamic interaction measures.

Solid line is for the control system with integral mode having the static decoupler, dashed line is for the multiloop control system for $G(s)$, and dotted line is for the multiloop control system for $G(s)G^{-1}(0)$.

similar sensitivity and complementary sensitivity function matrices, as expected in the design stage.

One of the dynamic interaction measures in Lee and Edgar[22] is defined as

$$q(\lambda) = \max_{\omega} \sigma_{\max}[H(j\omega) - \bar{H}(j\omega)]$$

$$\bar{H}(s) = \mathrm{diag}\left[\bar{h}_i(s) = \frac{g_{ii}^{+}(s)}{(\lambda s + 1)^{r_i}}\right] \quad (11)$$

which means the largest magnitude of the difference between the actual closed-loop transfer function matrix $H(s)$ and the desirable closed-loop transfer function matrix $\bar{H}(s)$. If $q(\lambda)$ is small, controller performances will not differ much from desirable control performances. This dynamic interaction measure can also be used to determine quantitatively whether the proposed controller 4 is better than controller 1. Figure 4 shows $q(\lambda)$ for the Wood–Berry column model 2. Equation 11 is applied to $G(s)$, $G(s)G^{-1}(0)$ with multiloop controllers 7 and $G(s)$ with controllers with integral mode having the static decoupler as

$$C(s) = \mathrm{diag}\left[\frac{1}{g_{ii}(s)}\frac{\bar{h}_i(s)}{1 - \bar{h}_i(s)} - \frac{k_{0i}}{s}\right] + \frac{1}{s}G^{-1}(0)\mathrm{diag}[k_{0i}g_{ii}(0)]$$

We can see that control systems with static decouplers have lower dynamic interaction measures when λ is large (processes are slowly controlled). The control system with integral mode having the static decoupler shows the best values of dynamic interaction measure.

Interaction measures are used to determine structures of multiloop control systems. Because interactions at low frequencies can be removed with the static decoupler added to the integral modes, input and output structures that are dynamically better but show high interactions at low frequencies may effectively be used for the proposed control system.

Stability Robustness and Loop Failure Tolerance

Robustness of a control system is very important because the process model can have uncertainties in its parameters. Stability robustness of the proposed controller is expected to be very similar to the multiloop controller because only low-frequency responses are different from each other. The detailed analysis of robustness is highly dependent on the uncertainty model. Here, to illustrate stability robustness of the proposed control system, several uncertainty models are considered. First, for a process input uncertainty as $G(s)[I + \Delta_I(s)]$, where $\Delta_I(s)$ is stable, the closed-loop system is stable if[2]

$$\|\Delta_I(j\omega)\| < 1/\sigma_{\max}\{[I + C(j\omega)G(j\omega)]^{-1}C(j\omega)G(j\omega)\} \quad (12)$$

It can be derived from the characteristic equation of

$$\begin{aligned} T(s) &= \det\{I + G(s)[I + \Delta_I(s)]C(s)\} = \det[I + G(s)C(s) \\ &\qquad + G(s)\Delta_I(s)C(s)] \\ &= \det[I + G(s)C(s)]\det\{I + [I + G(s)C(s)]^{-1}G(s)\Delta_I(s)C(s)\} \\ &= \det[I + G(s)C(s)]\det\{I + C(s)[I + G(s)C(s)]^{-1}G(s)\Delta_I(s)\} \\ &= \det[I + G(s)C(s)]\det\{I + [I + C(s)G(s)]^{-1}C(s)G(s)\Delta_I(s)\} \end{aligned} \quad (13)$$

By applying the above technique, we can also obtain the stability condition for a process output uncertainty as $[I + \Delta_O(s)]G(s)$, where $\Delta_O(s)$ is stable. The closed-loop system is stable if $\|\Delta_O(j\omega)\| < 1/\sigma_{\max}\{[I + G(j\omega)C(j\omega)]^{-1}G(j\omega)C(j\omega)\}$. Figure 5 shows these stability bounds for the Wood–Berry column model with controllers in Table 1. We can see that the proposed control system has the largest stability region of process input or output uncertainty.

When the inverse of the process steady-state gain model has

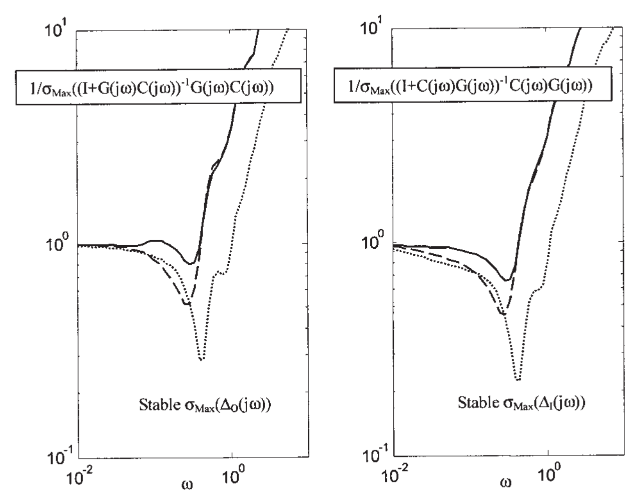

Figure 5. Stability regions of input and output uncertainties.

Solid line is for the control system with integral mode having the static decoupler, dashed line is for the multiloop PI control system designed by the desired closed-loop response method, and dotted line is for the multiloop control system designed by the biggest log-modulus tuning method.

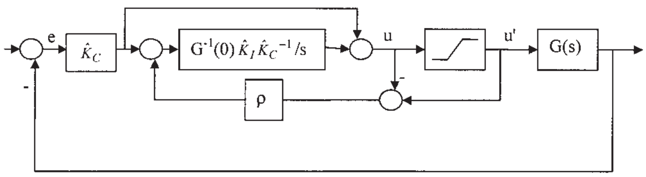

Figure 6. An antireset windup scheme for the proposed control system.

an error of $G^{-1}(0)(I + \Delta)$, where Δ is real, the proposed controller has an error of

$$\frac{1}{s}G^{-1}(0)(I+\Delta)\hat{K}_I + \hat{K}_C = C(s) + \frac{1}{s}G^{-1}(0)\Delta\hat{K}_I$$

and thus the control system maintains stability if

$$\|\Delta\| < \max_{\omega\in R} \mu_R\left\{\frac{1}{j\omega}\hat{K}_I[I + G(j\omega)C(j\omega)]^{-1}G(j\omega)G(0)^{-1}\right\}$$

where $\mu_R(M)$ is the real structured singular value[2,15] of M. For the full real matrix of Δ, $\mu_R(\cdot)$ can be calculated exactly.[19] For the control system of the Wood–Berry column and the proposed controller in Table 1, the size of uncertainty for which the proposed control system remains stable is $\|\Delta\| < 0.85$. The process gain matrix can have uncertainties of 85% of that to be singular.

Loop failure tolerance is another important condition for a multivariable control system to be satisfied because some inputs can be saturated as a result of large disturbances or opened for maintenance. It can be checked through a robust stability problem for the uncertainties in the process such as $G(s)(0.5I + \Delta)$, where Δ is a real diagonal.[2] If the stability radius for magnitude of Δ is >0.5, the closed-loop system will be loop failure tolerant. The computation of real stability radius or equivalent real structured singular value has been found to be very difficult,[15] and thus upper and lower bounds are often used. By adjusting the λ_i design parameters, we can make the proposed control system satisfy the condition of loop failure tolerance.

Antireset Windup Scheme

To reduce undesirable effects of integral terms when inputs have lower and upper limits, an antireset windup scheme is used. Loop failure tolerance is often automatically satisfied when an antireset windup scheme is used. Here an antireset windup scheme slightly modified from the conventional antiwindup scheme[20] is adopted (Figure 6). For the stability of feedback implementation of $C(s)$ as in Figure 6, it is assumed that eigenvalues of $G_m^{-1}(0)\hat{K}_{\mathrm{Im}}\hat{K}_{Cm}^{-1}$ are all in the open right-half-plane for every $G_m(0)$, a principal submatrix of $G(0)$, and $\hat{K}_{\mathrm{Im}}$ and $\hat{K}_{Cm}$ are the corresponding subsets of $\hat{K}_I$ and $\hat{K}_C$, respectively. Diagonally dominant processes satisfy this condition. This eigenvalue condition of a matrix can be found in the area of D-stability or the decentralized integral controllability.[21] If this assumption is not met, other types of antireset windup schemes should be considered.

Control performances of remaining loops are shown to be maintained whenever some inputs are opened. The transfer function matrix for $U(s)$ is

$$U(s) = \left[I + \rho\frac{1}{s}G^{-1}(0)\hat{K}_I\hat{K}_c^{-1}\right]^{-1} \times \left[I + \frac{1}{s}G^{-1}(0)\hat{K}_I\hat{K}_c^{-1}\right]\hat{K}_c E(s) + \left[I + \rho\frac{1}{s}G^{-1}(0)\hat{K}_I\hat{K}_c^{-1}\right]^{-1}\rho\frac{1}{s}G^{-1}(0)\hat{K}_I\hat{K}_c^{-1}U'(s) \quad (14)$$

If eigenvalues of $G^{-1}(0)\hat{K}_I\hat{K}_C^{-1}$ are all in the open right-half-plane, the closed-loop system is stable for any value of ρ and it can be chosen to be large enough so that the bandwidth of feedback system is about 10 times larger than that of the whole control system. Dynamic effects of the feedback implementation of $C(s)$ can be ignored with such ρ.

When all variables are within their limits, $U'(s) = U(s)$ and we have

$$U(s) = \left[\frac{1}{s}G^{-1}(0)\hat{K}_I + \hat{K}_c\right]E(s) \quad (15)$$

If some process inputs are fixed to their limits, the transfer function matrix between the inputs that are within their limits and the corresponding errors is approximately

$$U_m(s) \approx \left[\frac{1}{s}G_m^{-1}(0)\hat{K}_{\mathrm{Im}} + \hat{K}_{Cm}\right]E_m(s) \quad (16)$$

where $U_m(s)$ are the input variables that are within their limits, $E_m(s)$ are the error variables corresponding to $U_m(s)$, $G_m(0)$ is the steady-state gain matrix between $U_m(s)$ and $Y_m(s)$, and $\hat{K}_{\mathrm{Im}}$ and $\hat{K}_{Cm}$ are corresponding subsets of $\hat{K}_I$ and $\hat{K}_C$, respectively. Derivation of this relation is given in the Appendix. Because $\hat{K}_I$ and $\hat{K}_C$ depend only on the diagonal elements of $G(s)$, Eq. 16 is the same as the proposed control system designed for $G_m(s)$, a principal subsystem of $G(s)$. Closed-loop performance will be maintained whenever some input variables are kept at their limits or out of service.

As an illustration, consider a 2×2 process. Assume that the second process input is fixed at its limit. Then $U'(s) = [u_1(s)\ \ \xi_2(s)]^T$, where $\xi_2(s)$ is a constant variable representing the upper or lower bound of the second process input. Transfer functions are

$$\begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} = \begin{bmatrix} 1 & \frac{\rho}{s}q_{12} \\ 0 & 1+\frac{\rho}{s}q_{22} \end{bmatrix}^{-1} \begin{bmatrix} 1+\frac{1}{s}q_{11} & \frac{1}{s}q_{12} \\ \frac{1}{s}q_{21} & 1+\frac{1}{s}q_{22} \end{bmatrix} \times \begin{bmatrix} k_{C1} & 0 \\ 0 & k_{C2} \end{bmatrix}\begin{bmatrix} e_1(s) \\ e_2(s) \end{bmatrix} + \begin{bmatrix} 1 & \frac{\rho}{s}q_{12} \\ 0 & 1+\frac{\rho}{s}q_{22} \end{bmatrix}^{-1}\begin{bmatrix} \frac{\rho}{s}q_{12} \\ 1+\frac{\rho}{s}q_{22} \end{bmatrix}\xi_2(s)$$

$$Q = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} = G(0)^{-1}K_IK_C^{-1} \quad (17)$$

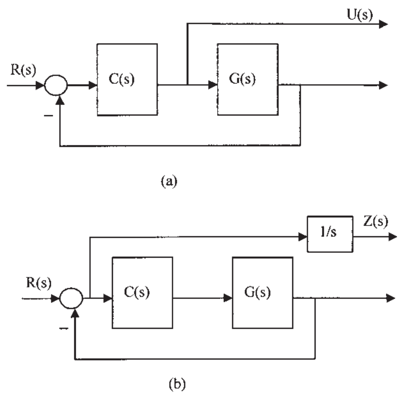

Figure 7. Models for the closed-loop identification of process steady-state gains.

Thus the transfer function between $e_1(s)$ and $u_1(s)$ is

$$\frac{u_1(s)}{e_1(s)} = \left[1 + \frac{q_{11}}{s} - \frac{\rho q_{12}q_{21}}{s^2\left(1 + \rho\dfrac{q_{22}}{s}\right)}\right]k_{C1}$$

$$\approx \left(1 + \frac{q_{11} - q_{12}q_{21}/q_{22}}{s}\right)k_{C1} = \left[k_{C1} + \frac{1}{s}\frac{k_{I1}}{g_{11}(0)}\right] \quad (18)$$

when ρ is sufficiently large.

Closed-Loop Identification of Process Steady-State Gains

For the static decoupler, the steady-state process gains are required. They can be identified from closed-loop step responses. Consider the control system of Figure 7a. The closed-loop transfer function matrix between $R(s)$ and $U(s)$ is $U(s) = C(s)[I + G(s)C(s)]^{-1}R(s)$. Because $C(s)$ has invertible integral gains, we have

$$U(0) = G^{-1}(0)R(0) \quad (19)$$

and we can obtain $G^{-1}(0)$ from step changes of the set points.

If the control inputs $u(t)$ are noisy, we can use the system of Figure 7b where error variables are integrated. Because $Z(s) = (1/s)\{I - [I + G(s)C(s)]^{-1}G(s)C(s)\}R(s) = (1/s)[I + G(s)C(s)]^{-1}R(s)$, the steady-state gain relations are

$$Z(0) = \hat{K}_I^{-1}G^{-1}(0)R(0) \quad (20)$$

Because the integral gain $\hat{K}_I$ is known, we can obtain $G^{-1}(0)$ from the steady-state gains between $z(t)$ and $r(t)$. From each step response, we can obtain each column of $G^{-1}(0)$.

Structure Selection Criterion and Design Procedure

Interaction measures such as relative gain array (RGA)[12] and dynamic interaction measure[22] can be used to select the structure of control systems with static decouplers. Consider the RGA number[15]

$$\text{RGA number} \equiv \|R[G(j\omega)] - I\| \quad (21)$$

This RGA number is a measure for diagonal dominance and is often used for pairings of multiloop control systems. Detailed RGA conditions for multiloop controls can be found in Skogestad and Postlethewaite.[15] Qualitatively, if the RGA number for $G(j\omega)$ is small for a range of frequencies covering the bandwidth of a closed-loop system {that is, $R[G(j\omega)] \approx I$}, a multiloop control system without static decoupler will be sufficient. Otherwise, inclusion of a decoupler is considered.

The RGA number for $G(j\omega)G^{-1}(0)$ is 0 at $\omega = 0$. If the RGA number for $G(j\omega)G^{-1}(0)$ is not greater than that of $G(j\omega)$ and remains small for the required range of frequencies, the static decoupler is applied to the whole controller (controller 1). On the other hand, if the RGA number for $G(j\omega)G^{-1}(0)$ becomes greater than that of $G(j\omega)$ as the frequency increases, the static decoupler is applied only to the integral mode (controller 4). For processes whose diagonal transfer functions have smaller time constants than off-diagonal transfer functions, the RGA numbers for $G(j\omega)G^{-1}(0)$ usually grow as the frequency increases and the proposed controller 4 will be selected. Other interaction measures can also be used for the control structure selection in a similar manner.

It is noted that the RGA number for $G(j\omega)G^{-1}(0)$ is small when the frequency is not high. By adjusting the bandwidth of control system, we can make the RGA number for $G(j\omega)G^{-1}(0)$ remain small for the whole required range of frequencies. Thus, a control system with static decoupler having a desired robustness and performance can always be designed at the expense of control system speed.

In summary, the proposed control system with the static decoupler can be designed as follows.

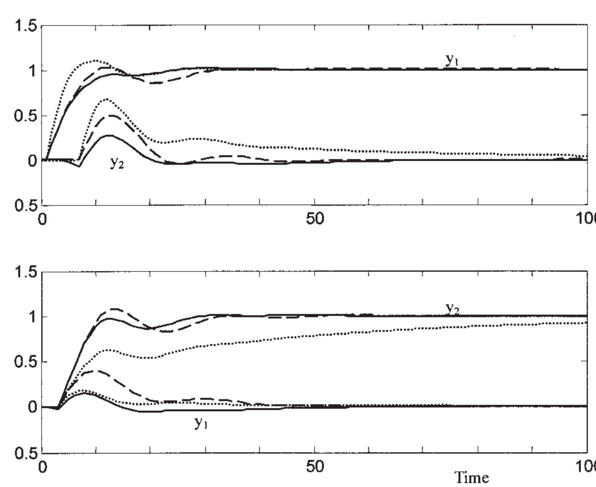

Figure 8. Step set-point responses for control systems of Wood–Berry column example.

Solid line is for the proposed control system with integral mode having the static decoupler, dashed line is for the multiloop control system tuned by desired closed-loop response method, and dotted line is for the multiloop control system tuned by the biggest log-modulus tuning method.

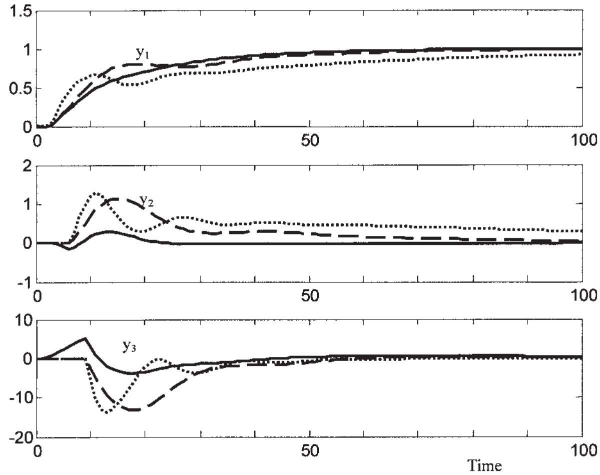

Figure 9. Step set-point responses for control systems of Ogunnaike–Ray column example.

Solid line is for the proposed control system with integral mode having the static decoupler, dashed line is for the multiloop control system tuned by desired closed-loop response method, and dotted line is for the multiloop control system tuned by the biggest log-modulus tuning method.

Step 1. Applying interaction measures such as RGA and the decentralized integral controllability,[15] select the structure of multiloop control systems.

Step 2. Applying RGA or other dynamic interaction measure, select the better control structure with a static decoupler, either controller 1 or 4.

Step 3. When the proposed controller 4 having integral mode with the static decoupler is chosen, design and implement the proposed control system with the antireset windup scheme. If control performances are poor, field-tune it by finding the process steady-state gains. On the other hand, when controller 1 is chosen, design the control system with controller 1 by applying multiloop controller design methods to $G(s)G^{-1}(0)$.

Examples

To illustrate control performances of proposed control system, two examples are considered: the Wood–Berry column model of Eq. 2 and the Ogunnaike–Ray column model,[24] expressed in the following equation

$$G(s) = \begin{bmatrix} \dfrac{0.66\ \exp(-2.6s)}{6.7s+1} & \dfrac{-0.61\ \exp(-3.5s)}{8.64s+1} & \dfrac{-0.0049\ \exp(-s)}{9.06s+1} \\ \dfrac{1.11\ \exp(-6.5s)}{3.25s+1} & \dfrac{-2.36\ \exp(-3s)}{5s+1} & \dfrac{-0.01\ \exp(-1.2s)}{7.09s+1} \\ \dfrac{-34.68\ \exp(-9.2s)}{8.15s+1} & \dfrac{46.2\ \exp(-9.4s)}{10.9s+1} & \dfrac{0.87(11.61s+1)\exp(-s)}{(3.89s+1)(18.8s+1)} \end{bmatrix} \quad (22)$$

The proposed control system is compared with multiloop control systems designed by the analytic desired closed-loop response method[17] and the biggest log-modulus tuning method.[24] Tuning results are given in Table 1. Figures 8 and 9 show step set-point responses. We can see that, compared to the multiloop control system designed by the biggest log-modulus tuning method, the closed-loop responses are improved by tuning with the desired closed-loop response method and furthermore by implementing the static decoupler on the integral modes. Also the proposed control system having integral modes with the static decoupler is less oscillatory and substantially suppresses excursions of nonpaired outputs for step set-point changes.

Conclusions

It is shown that static decouplers should be applied only to the integral modes for processes where usual static decouplers introduce undesirable loop interactions at high frequencies. While maintaining the simplicity of multiloop control systems, we can improve closed-loop performances. For online tuning of the proposed control system, a closed-loop identification method to find process steady-state gains is also proposed. A conventional antireset windup scheme is adopted to reduce undesirable effects of integral term when process inputs have physical limitations. It is shown that the closed-loop performance is maintained when some process inputs are fixed at their limits or out of service.

Frequency responses of sensitivity and complementary sensitivity function matrices illustrate that the proposed static decoupler implemented on the integral mode has decoupled responses at low frequencies for performance and multiloop responses at high frequencies for robust stability, showing lower peak amplitude ratios and wider bandwidths than those of multiloop control systems. As expected, time-domain simulations show that the proposed control systems are less oscillatory and suppress excursions of nonpaired outputs for step set-point changes.

Acknowledgments

The first author acknowledges support from the Basic Research Program of the Korea Science & Engineering Foundation under Grant R05-2003-000-11208-0.

Literature Cited

1. Koivo HN, Pohjolainen S. Tuning of multivariable PI-controllers for unknown systems with input delay. *Automatica*. 1985;21:81-91.
2. Maciejowski JM. *Multivariable Feedback Design*. New York, NY: Addison–Wesley; 1989.
3. Tan W, Marquez HJ, Chen T. Multivariable robust controller design

for a boiler system. *IEEE Trans Control Syst Technol.* 2002;10:735-742.
4. Luyben WL. Distillation decoupling. *AIChE J.* 1970;16:198-203.
5. Waller KVT. Decoupling in distillation. *AIChE J.* 1974;20:592-594.
6. Shinskey FG. *Process Control Systems: Application, Design and Adjustment*. New York, NY: McGraw-Hill; 1988.
7. Seborg DE, Edgar TF, Mellichamp DA. *Process Dynamics and Control*. 2nd Edition. New York, NY: Wiley; 2004.
8. Arkun Y, Manousouthakis B, Palazoglu A. Robustness analysis of process control systems: A case study of decoupling control in distillation. *Ind Eng Chem Process Des Dev*. 1984;23:93-101.
9. Wade HL. Inverted decoupling: a neglected technique. *ISA Trans*. 1970;36:3-10.
10. Waller M, Waller JB, Waller KV. Decoupling revisited. *Ind Eng Chem Res Dev*. 2003;42:4575-4577.
11. Astrom KJ, Johansson KH, Wang QG. Design of decoupled PI controllers for two-by-two systems. *IEEE Proc Control Theory Appl*. 2002;149:74-81.
12. Bristol EH. On a new measure of interaction for multivariable process control. *IEEE Trans Autom Control*. 1966;AC-11:133-134.
13. Wood RK, Berry MW. Terminal composition control of a binary distillation column. *Chem Eng Sci*. 1973;28:1707-1717.
14. Skogestad S, Morari M. Implications of large RGA elements on control performance. *Ind Eng Chem Fundam*. 1987;26:2323-2330.
15. Skogestad S, Postlethewaite I. *Multivariable Feedback Control*. New York, NY: Wiley; 1996.
16. Lee J, Edgar TF. Improved PI controller with delayed or filtered integral mode. *AIChE J*. 2002;48:2844-2850.
17. Lee M, Lee K, Kim C, Lee J. Analytic design of multiloop PID controllers for desired closed-loop responses. *AIChE J*. 2004;50:1631-1635.
18. Lee Y, Park S, Lee M, Brosilow C. PID controller tuning for desired closed-loop responses for SI/SO systems. *AIChE J*. 1998;44:106-115.
19. Qiu L, Bernhardsson B, Rantzer A, Davison EJ, Young PM, Doyle JC. A formula for computation of the real stability radius. *Automatica*. 1995;31:879-890.
20. Kothare MV, Campo PJ, Morari M, Nett CN. A unified framework for the study of anti-windup designs. *Automatica*. 1994;30:1869-1883.
21. Campo PJ, Morari M. Achievable closed-loop properties of systems under decentralized control: conditions involving the steady-state gains. *IEEE Trans Autom Control*. 1994;39:932-943.
22. Lee J, Edgar TF. Dynamic interaction measures for decentralized control of multivariable processes. *Ind Eng Chem Res Dev*. 2004;43:283-287.
23. Ogunnaike BA, Ray WH. Multivariable controller design for linear systems having multiple time delays. *AIChE J*. 1979;25:1043.
24. Luyben WL. Simple method for tuning SISO controllers in multivariable systems. *Ind Eng Chem Process Des Dev*. 1986;25:654-660.

Appendix

Without loss of generality, the first m inputs are assumed to be within their limits and other inputs are fixed at their limits or in open loop. Then we have

$$U'(s) = XU(s) + \xi(s)$$

$$X = \begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix} \tag{A1}$$

where $\xi(s)$ represents the variables fixed to their limits. Substituting this into Eq. 14 and removing terms about $\xi(s)$,

$$\begin{aligned} U(s) &= \left[I - \left(I + \frac{\rho Q}{s} \right)^{-1} \frac{\rho Q X}{s} \right]^{-1} \left(I + \frac{\rho Q}{s} \right)^{-1} \\ &\qquad \times \left(I + \frac{Q}{s} \right) \hat{K}_c E(s) \\ &= [sQ^{-1} + \rho(I - X)]^{-1}(sQ^{-1} + I)\hat{K}_c E(s) \end{aligned} \tag{A2}$$

where $Q = G^{-1}(0)\hat{K}_I\hat{K}_C^{-1}$. If ρ goes to infinity, we have

$$\begin{aligned} U(s) &= \begin{bmatrix} sQ_{11} & sQ_{12} \\ sQ_{21} & sQ_{22} + \rho I_{n-m} \end{bmatrix}^{-1} \\ &\qquad \times \begin{bmatrix} sQ_{11} + I_m & sQ_{12} \\ sQ_{21} & sQ_{22} + I_{n-m} \end{bmatrix} \hat{K}_C E(s) \\ &\approx \begin{bmatrix} \frac{1}{s} Q_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} sQ_{11} + I_m & sQ_{12} \\ sQ_{21} & sQ_{22} + I_{n-m} \end{bmatrix} \hat{K}_c E(s) \\ &= \begin{bmatrix} \frac{1}{s} Q_{11}^{-1} + I_m & Q_{11}^{-1} Q_{12} \\ 0 & 0 \end{bmatrix} \hat{K}_c E(s) \end{aligned} \tag{A3}$$

where

$$Q = \hat{K}_C \hat{K}_I^{-1} G(0) = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}$$

and $Q_{11} = \hat{K}_{Cm}\hat{K}_{\mathrm{Im}}^{-1}G_m(0)$. $U_m(s)$ are the input variables that are within their limits, $E_m(s)$ are the error variables corresponding to $U_m(s)$, $G_m(0)$ is the steady-state gain matrix between $U_m(s)$ and $Y_m(s)$, and $\hat{K}_{\mathrm{Im}}$ and $\hat{K}_{Cm}$ are subsets of $\hat{K}_I$ and $\hat{K}_C$, respectively. Thus

$$\begin{aligned} U_m(s) &\approx \left[I + \frac{1}{s} G_m^{-1}(0) \hat{K}_{Im} \hat{K}_{Cm}^{-1} \right] \hat{K}_{Cm} E_m(s) \\ &= \left[\frac{1}{s} G_m^{-1}(0) \hat{K}_{Im} + \hat{K}_{Cm} \right] E_m(s) \end{aligned} \tag{A4}$$

This is the same as the proposed controller applied to the process $G_m(s)$.

Manuscript received Aug. 2, 2004, and revision received Feb. 3, 2005.